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A Theoretical Study of Pressure Drop for Non-Newtonian Creeping Flow Past an Assemblage of Spheres

A combination of Happel's free surface model and variational principles is used to obtain bounds on the drag offered by the creeping flow of a power law fluid past an assemblage of solid spheres. The theoretical predictions of the product of the Fanning friction factor f and Reynolds number Re_p are in close agreement with available experimental data on non-Newtonian flow through porous media. The product ($f\ Re_p$) reduces for the Newtonian case to that of Happel and Brenner.

V. MOHAN

and

J. RAGHURAMAN

Department of Chemical Engineering
Indian Institute of Technology
Madras 600 036, India

SCOPE

The flow of a Newtonian fluid through packed and fluidized beds has received considerable attention in the chemical engineering literature. A free surface model developed by Happel is widely used to predict the friction factor for Newtonian fluids. However, the analogous problem of flow of a non-Newtonian fluid through packed and fluidized beds has not been analyzed so far with this model. This paper extends the Happel's free surface model to the flow of power law fluids by making use of variational principles and presents an analysis for the friction factor in packed and fluidized beds.

CONCLUSIONS AND SIGNIFICANCE

A combination of Happel's free surface model and variational principles yields numerical values for the product of the Fanning friction factor f and the Reynolds number Re_p for various values of bed porosities and flow behavior indexes. An expression for the product ($f\ Re_p$) in terms of the porosity and flow behavior index is developed which well predicts experimental values of friction factor for power law flow through packed and fluidized beds.

V. Mohan is with the Department of Chemical Engineering, Illinois Institute of Technology, Chicago, Illinois 60616. J. Raghuraman is with the Department of Chemical Engineering, Monash University, Clayton, Victoria 3168, Australia.

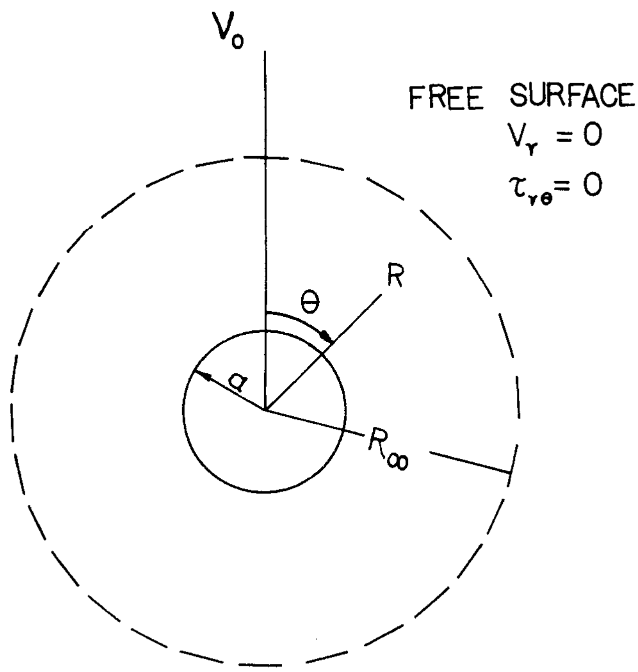

Fig. 1. Schematic diagram of the flow system.

Fluid-particle systems with a large volume fraction of solids have not been satisfactorily modeled, the main difficulties being an adequate representation of particle-particle interaction, wall effects, and/or entry and exit effects. One well-known model which seeks to surmount these problems in part is the cell model of Happel (1958). In this model, wall effects and/or entry and exit effects are neglected, and the particles in the assembly are assumed to be spherical and equally spaced in the radial and longitudinal directions. The interaction of the neighboring spheres with any individual sphere is modeled by assuming that the sphere is surrounded by a hypothetical spherical interface uncoupled from the other spheres. The bed voidage and the particle interaction are imposed on the cell by the position of the outer spherical boundary and by imposing a zero shear stress and a zero radial velocity on the hypothetical surface. The sphere in each unit cell moves with a velocity equal to the superficial velocity of the fluid in the assembly. The cell is conceived to represent the total constraining effect of the neighboring particles, thus accounting for the particle interactions. This is the free surface model.

By using the above model, the prediction of pressure drop and/or hindered settling in concentrated systems of spheres properly reduces to Stoke's law at infinite dilution. The model fails to predict hindered settling of sedimenting solutions in the dilute range and overpredicts the drag in fluidized beds, but for packed beds in the range of voidages $0.3 < \epsilon < 0.6$, it predicts with remarkable success the pressure drop at low particle Reynolds numbers (Happel and Brenner, 1973; El-Kaissy and Homsy, 1973). Therefore, one area in which cell models appear promising is in the description of steady flow through fluid-particle assemblages with voidages in the packed bed range.

A similar model of Kuwabara (1959) which imposes a zero vorticity on the hypothetical surface is in conceptual error, for the cell does work on the surroundings. Thus, the cell model of Happel is more sound for the analysis of flow situations in fluid-particle systems.

The solution for the creeping flow of a Newtonian fluid past an assemblage of spheres was first given by Happel (1958) and for the intermediate Reynolds numbers regime by Le Clair and Hamielec (1968) using finite-difference methods and El-Kaissy and Homsy (1973) using a regular perturbation with the Reynolds number as the perturbation parameter. The creeping flow of a power law non-Newtonian fluid past a single solid sphere has been theoretically investigated by Tomita (1959), Wallick et al. (1962), Slattery (1962), and Wasserman and Slattery (1964) using variational principles. The intermediate Reynolds number flow was analyzed by Adachi et al. (1973) using an extended Galerkin method.

The reported experimental studies on non-Newtonian flow through porous media and packed beds are limited (Savins, 1969), and a theoretical understanding of this flow situation has not so far been attempted. This work develops the bounds on the drag during the creeping flow of a power law fluid past an assemblage of solid spheres by using a combination of Happel's free surface model and the variational principles of Slattery (1972). The predictions of the theory are then compared with available experimental data in porous media in the creeping flow regime.

STATEMENT OF THE PROBLEM

Consider the steady, incompressible, creeping flow of a power law fluid past an assemblage of insoluble solid spheres (Figure 1). The internal sphere moves in the direction of the positive Z axis with a velocity V_o (equal to the superficial velocity of the fluid in the assemblage) inside a hypothetical fluid sphere on which the radial velocity and the tangential stress vanish. The equation of continuity and the equation of motion are given by

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

and

$$\nabla \cdot \mathbf{T} + \rho \mathbf{f} = 0 \tag{2}$$

By using the spherical polar coordinates, the boundary conditions on the surface of the sphere are given by

$$v_r = V_o z, \quad \text{at} \quad r = 1$$

$$v_\theta = -V_o (1 - z^2)^{1/2}, \quad \text{at} \quad r = 1 \tag{3}$$

where

$$z = \cos\theta \tag{4}$$

The boundary conditions on the free surface are

$$v_r = 0, \quad \text{at} \quad r = r_\infty, \tag{5a}$$

$$\tau_{r\theta} = 0 \quad \text{or} \quad d_{r\theta} = 0, \quad \text{at} \quad r = r_\infty \tag{5b}$$

where r_∞ is the dimensionless radius of the cell related to the voidage of the multiparticle assemblage by the expression

$$r_\infty = R_\infty / a = (1 - \epsilon)^{-1/3} \tag{6}$$

where R_∞ is the radius of the hypothetical fluid envelope, and ϵ is the cell voidage. The quantity $d_{r\theta}$ is the component of the rate-of-deformation tensor.

By using the definition of $d_{r\theta}$, Equation (5) can be written as

$$v_r = 0, \text{at } r = r_\infty$$

$$\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) = 0, \quad \text{at} \quad r = r_\infty \tag{7}$$

For the above flow system, the energy dissipation is wholly within the hypothetical fluid cell enveloping the solid sphere, as the cell does no work on its surroundings.

ANALYSIS

For the steady creeping flow of an incompressible fluid, two variational principles were given by Slattery (1972):

$$\int_V E\, dV \leq \int_V E^* \, dV - \int_{S-S_v} (\mathbf{v}^* - \mathbf{v}) \cdot ([\mathbf{T} - \rho \Phi I] \cdot \mathbf{n})\, dS \tag{8}$$

and

$$\int_V E\, dV \geqq -\int_V E_c^*\, dV + \int_S \mathbf{v} \cdot ([T^* - \rho\,\Phi\, I]\cdot n)\, dS \quad (9)$$

where the work function E and the complementary work function E_c are defined by

$$E = \int_0^{II} \eta\, dII \quad (10)$$

and

$$E_c = \int_0^{II_\tau} \frac{dII_\tau}{4\eta} \quad (11)$$

In inequalities (8) and (9), the volume domain V for integration is the flow domain and the surface S is the surface bounding the flow domain. Inequality (8) is called the velocity variational principle, and the quantities with a superscript asterisk are evaluated on the basis of a trial velocity profile that satisfies the equation of continuity and prescribed conditions for velocity on the part S_v of the bounding surface S on which the velocity is explicitly specified. Inequality (9) states the stress principle, and the quantities with a superscript asterisk are evaluated on the basis of a trial stress profile that satisfies Cauchy's first law and the prescribed conditions for stress on the part S_t of the bounding surface on which the stress is explicitly specified.

For the flow of a power law fluid with flow behavior index n, the energy dissipation rate per unit volume is related to the work function by (Slattery, 1972):

$$tr\,(\boldsymbol{\tau}\cdot\nabla\mathbf{v}) = (n+1)\, E \quad (12)$$

Equation (12) and inequalities (8) and (9) can be used to set up bounds on the energy dissipation rate in the unit cell.

The bounding surface S is the surface of the solid sphere and the free surface. On the former, the velocity is specified and, therefore, $(S - S_v)$ is the free surface. For a trial velocity profile which is chosen to satisfy $v_r = 0$ on the free surface, it can be shown by using Equation (5) that

$$\int_{(S-S_v)} (\mathbf{v}^* - \mathbf{v}) \cdot ([\mathbf{T} - \rho\,\Phi\, I]\cdot n)\, dS = 0 \quad (13)$$

Furthermore, for a trial stress profile satisfying $\tau^*_{r\theta} = 0$ on the free surface, it is seen from Equation (5a) that

$$\int_{(S-S_v)} \mathbf{v}\cdot([\mathbf{T}^* - \rho\,\Phi\,\mathbf{I}]\cdot n)\, dS = 0 \quad (14)$$

Inequalities (8) and (9) together with Equations (13) and (14) combine to yield

$$(n+1)\int_V E^*\, dV \geqq \left[\mathcal{E} = \int_V tr\,(\tau\cdot\nabla\mathbf{v})\, dV = (n+1)\int_V E\, dV\right] \geqq (n+1)\left\{-\int_V E_c^*\, dV + \int_{S(r=1)} \mathbf{v} \cdot ([\mathbf{T}^* - \rho\,\Phi\, I]\cdot\mathbf{n})\, dS\right\} \quad (15)$$

Inequality (15) gives the upper and lower bounds on the energy dissipation rate $\mathcal{E}$.

Upper bound

Since the flow is axisymmetric, $v_\phi = 0$ and a dimensionless stream function ψ is defined such that

$$\frac{v_r}{V_o} = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad \frac{v_\theta}{V_o} = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \quad (16)$$

A trial stream function profile is chosen in the form

$$\psi^* = \left(A_1 r^2 + A_2 r^\sigma + \frac{A_3}{r} + A_4 r^4\right)(1 - z^2) \quad (17)$$

With $\sigma = 1$, Equation (17) reduces to the form for the Newtonian flow situation. By using Equations (3), (7), (16), and (17), it can be shown that

$$A_1 + A_2 + A_3 + A_4 = -1/2$$
$$2A_1 + \sigma A_2 - A_3 + 4A_4 = -1$$
$$r_\infty^3 A_1 + r_\infty^{\sigma+1} A_2 + A_3 + r_\infty^5 A_4 = 0$$
$$(\sigma - 1)(\sigma - 2) r_\infty^{\sigma+1} A_2 + 6A_3 + 6 r_\infty^5 A_4 = 0 \quad (18)$$

The constitutive equation for the power law fluid is given by

$$\boldsymbol{\tau} = 2\, K\, (2\, II)^{(n-1)/2}\, \mathbf{D} \quad (19)$$

The work function E defined by Equation (10) yields

$$E = \frac{K}{(n+1)}\,(2\, II)^{(n+1)/2} \quad (20)$$

Therefore, the first of the inequality (15) gives

$$V_o F_d = \mathcal{E} \leqq K \int_V (2\, II^*)^{(n+1)/2}\, dV \quad (21)$$

where the drag force F_d is related to the drag coefficient C_d by

$$F_d = C_d\, (\pi a^2)(1/2\, \rho\, V_o^2) \quad (22)$$

Equation (22) and inequality (21) can be combined and written in a dimensionless form to yield

$$Y_P = \frac{C_d\, Re_p}{24} \leqq \frac{2^{n-1}}{3}\, J_1 \quad (23)$$

where

$$J_1 = \int_{-1}^{1}\int_{x_\infty}^{1} (2\, \overline{II}^*)^{(n+1)/2}\, x^{-4} dx\, dz \quad (24)$$

and $\overline{II}^*$ is the dimensionless second invariant of the rate-of-deformation tensor for the trial stream function profile given by

$$\overline{II}^* = 6 z^2 [(2-\sigma) A_2 x^{3-\sigma} + 3 A_3 x^4 - 2 A_4/x]^2 + (1 - z^2)[(\sigma - 1)(\sigma - 2) A_2 x^{3-\sigma} + 6 A_3 x^4 + 6 A_4/x]^2/2 \quad (25)$$

From Equations (18) and (23) through (25), it is seen that the right side of inequality (23) is a function of σ alone, the minimum of which gives the upper bound Y_{UB}.

Lower bound

For the power law fluid, the definition of E_c given by Equation (11) and the constitutive equation given by Equation (19) yield

$$E_c = K^{-1/n}\,\frac{n}{n+1}\left(\frac{II_\tau}{2}\right)^{(n+1)/2n} \quad (26)$$

Trial extra stress profiles that reduce to the Newtonian case are chosen in the form

$$\tau^*_{rr} = -[K(V_o/a)^n](C x^D + C'x^B + C''/x)z$$
$$\tau^*_{\theta\theta} = -[K(V_o/a)^n](E x^D + E'x^B + E''/x)z \quad (27)$$
$$\tau^*_{\phi\phi} = -[K(V_o/a)^n](F x^D + F'x^B + F''/x)z$$
$$\tau^*_{r\theta} = -[K(V_o/a)^n](A'x^B + A''/x)(1-z^2)^{1/2}$$

where

$$x = 1/r \quad (28)$$

Since the pressure p is defined as

$$p = -\frac{1}{3} tr\,[\mathbf{T}]$$

and the elements τ_{ij} of the extra stress tensor are related to the elements T_{ij} of the stress tensor by

$$\tau_{ij} = T_{ij} - p\delta_{ij} \quad (29)$$

it is required that

$$tr\,[\boldsymbol{\tau}] = 0 \quad (30)$$

Therefore, from Equation (27)

$$C + E + F = 0$$
$$C' + E' + F' = 0 \quad (31)$$
$$C'' + E'' + F'' = 0$$

Furthermore, by substituting the trial profiles given by (27) into the equation of motion and by equating

$$\frac{\partial^2(p + \rho\,\Phi)^*}{\partial x\,\partial z} = \frac{\partial^2(p + \rho\,\Phi)^*}{\partial z\,\partial x} \quad (32)$$

it can be shown that

$$E = F$$
$$E' = F'$$
$$E'' = F''$$
$$D = 2$$
$$A' = -3F'/(B-1)$$
$$F'' = 2A''/3 \quad (33)$$

Using the condition of zero shear stress on the free surface, we get

$$A'' = -A'\,x_\infty^{B+1} \quad (34)$$

Equations (31), (33), and (34) express ten of the constants appearing in Equation (27) in terms of F, F' and B. These are chosen to maximize the right side of inequality (15).

From the θ component of the equation of motion, it can be shown that

$$\frac{(p+\rho\Phi)^*}{K(V_o/a)^n} = [-Fx^2 + \{(3-B)A' - F'\}\,x^B$$
$$+ (4A'' - F'')/x]z + C_o(x) \quad (35)$$

where $C_o(x)$ is some function of x. By combining Equations (3), (27), (31), and (33) through (35), the surface integral term in inequality (15) becomes

$$\int_{S(r=1)} \mathbf{v}\cdot([\mathbf{T}^* - \rho\Phi\mathbf{I}]\cdot n)\,dS = 4\pi K V_o^{n+1}\,a^{2-n}\,F \quad (36)$$

Furthermore, from Equation (26) we have

$$\int_V E_c^*\,dV = \pi K V_o^{n+1}\,a^{2-n}\,2^{(n-1)/2n}\frac{n}{n+1}J_2 \quad (37)$$

where

$$J_2 = \int_{-1}^{1}\int_{x_\infty}^{1}(\overline{II}^*_\tau)^{(n+1)/2n}\,x^{-4}\,dx\,dz \quad (38)$$

By using the trial extra stress profile given by Equation (27), it follows that

$$\overline{II}^*_\tau = 6z^2\,(Fx^2 + F'x^B + F''/x)^2$$
$$+ (1-z^2)(A'x^B + A''/x)^2/2 \quad (39)$$

Inequality (15) and Equations (22), (36), and (37) combine to yield

$$V_oF_d = \mathcal{E}$$
$$\geq (n+1)\,\pi\,KV_o^{n+1}\,a^{2-n}\left[-2^{(n-1)/2n}\frac{n}{n+1}J_2 + 4F\right] \quad (40)$$

In a dimensionless form, inequality (40) can be written to give

$$Y_P = \frac{C_d\,Re_p}{24}$$
$$\geq n+1\left[-\frac{2^{(2n^2-3n-1)/2n}}{3}\left(\frac{n}{n+1}\right)J_2 + \frac{2^nF}{3}\right] \quad (41)$$

The maximum of the right side of inequality (41) gives the lower bound Y_{LB} on Y_P.

COMPUTATIONAL RESULTS

The upper bound on Y_P was evaluated by minimizing the right side of inequality (23) by using a Fibonacci search (Mangasrian, 1972) on σ. For each value of σ in the search, values of A_1 to A_4 were evaluated by solving the system of Equations (18).

The lower bound on Y_P was obtained by maximizing the right side of inequality (41) by using a Rosenbrock search (Rosenbrock and Storey, 1966) on F, F', and B. For each set of values of F, F', and B, Equations (31), (33), and

TABLE 1. UPPER AND LOWER BOUNDS OF Y_P AS A FUNCTION OF ϵ AND n

	$n = 0.5$		$n = 0.6$		$n = 0.7$		$n = 0.8$		$n = 0.9$		$n = 1.0$
ϵ	Y_{UB}	Y_{LB}	Y_{UB}	Y_{LB}	Y_{UB}	Y_{LB}	Y_{UB}	Y_{LB}	Y_{UB}	Y_{LB}	$Y_{UB} = Y_{LB}$
0.4	10.59	10.37	16.08	15.92	24.43	24.32	37.06	37.0	56.18	56.04	85.12
0.5	6.41	6.34	9.17	9.11	13.09	12.91	18.67	18.5	26.61	26.48	37.91
0.6	4.24	4.19	5.73	5.69	7.73	7.69	10.42	10.39	14.04	14.02	18.92
0.7	2.98	2.94	3.81	3.78	4.87	4.85	6.22	6.21	7.94	7.91	10.13
0.8	2.19	2.15	2.65	2.62	3.2	3.19	3.87	3.86	4.67	4.67	5.64
0.9	1.68	1.64	1.9	1.88	2.15	2.14	2.44	2.43	2.75	2.75	3.11
0.99	1.44	1.33	1.45	1.39	1.46	1.43	1.47	1.46	1.47	1.47	1.48

TABLE 2. VALUES OF CONSTANTS IN EQUATIONS (44) AND (45)

i	0	1	2	3	4	5
a_i	7.1776	−8.3853	6.2055	−2.7352	0.6163	−0.0530
b_i	3.439	−4.0786	2.7426	−1.2734	0.3034	−0.0276

(34) determine the rest of the constants required for the evaluation of the integral in inequality (41). The integrals in the upper and lower bound computations were evaluated by using a two-dimensional Simpson's rule quadrature.

The upper and lower bounds on Y_P are given in Table 1 for various values of the voidage ϵ and the flow behavior index n. It is seen that for the form of the trial profiles chosen, remarkably close bounds on Y_P are obtained. For $n = 1$, the results reduce to those given by Happel and Brenner (1973).

It is useful to represent the results for the bounds on Y_P as the bounds on $(f \,.\, Re_p)$, where f is the Fanning friction factor. It can be shown that

$$f \,.\, Re_p = Y_P \cdot 9\,(1 - \epsilon) \qquad (42)$$

The arithmetic averages of the bounds on $(f \,.\, Re_p)$ are plotted in Figure 2, as the bounds are too close to be shown distinctly. It can be seen from the figure that the plot of log $(f \,.\, Re_p)$ is linear in the flow behavior index n and hence can be represented in the form

$$\ln\,(f \,.\, Re_p) = Q_1\, n + Q_2 \qquad (43)$$

where Q_1 and Q_2 are functions of ϵ. Fitting a Forsythe orthogonal polynomial (Carnahan, Luther, and Wilkes, 1969) to express Q_1 and Q_2 as functions of the voidage, we have

$$Q_1 = \sum_{i=0}^{5} a_i\,[-\ln\,(1-\epsilon)]^i \qquad (44)$$

$$Q_2 = \sum_{i=0}^{5} b_i\,[-\ln\,(1-\epsilon)]^i \qquad (45)$$

The values of a_i and b_i are listed in Table 2.

COMPARISON WITH EXPERIMENTAL DATA

The experimental data on pressure drop for non-Newtonian creeping flow in packed and fluidized beds are limited. Christopher and Middleman (1965), Yu et al. (1968), and Kemblowski and Mertl (1974) analyzed their data for power law flow in a packed and a fluidized bed using a capillary flow approach incorporating a correction factor for the tortuosity in the flow path. Aqueous solutions of carboxymethyl cellulose, polyethylene oxide, polyacrylamide (separan), and polyisobutylene in toluene were used by these investigators. Typical experimental data of the above investigators in the creeping flow regime recalculated on the basis of the definitions of Re_p and f used in the present analysis are compared in Figure 3 with the values of $(f \,.\, Re_p)$ predicted from the present theory. The values of the flow behavior index and bed voidage are also given in Figure 3. It is seen that the cell model successfully predicts the pressure drop during the creeping flow of power law fluids in multiparticle systems over a wide range of flow behavior index. The comparison could be made in a limited range of porosities due to lack of experimental data in the creeping flow regime.

The results of the present work indicate that a combination of the Happel's free surface model and variational principles is an excellent alternative approach to the capillary model in analyzing the creeping flow of non-Newtonian fluids in particle assemblages. This also forms the first step for the analysis of flow in complicated geometries.

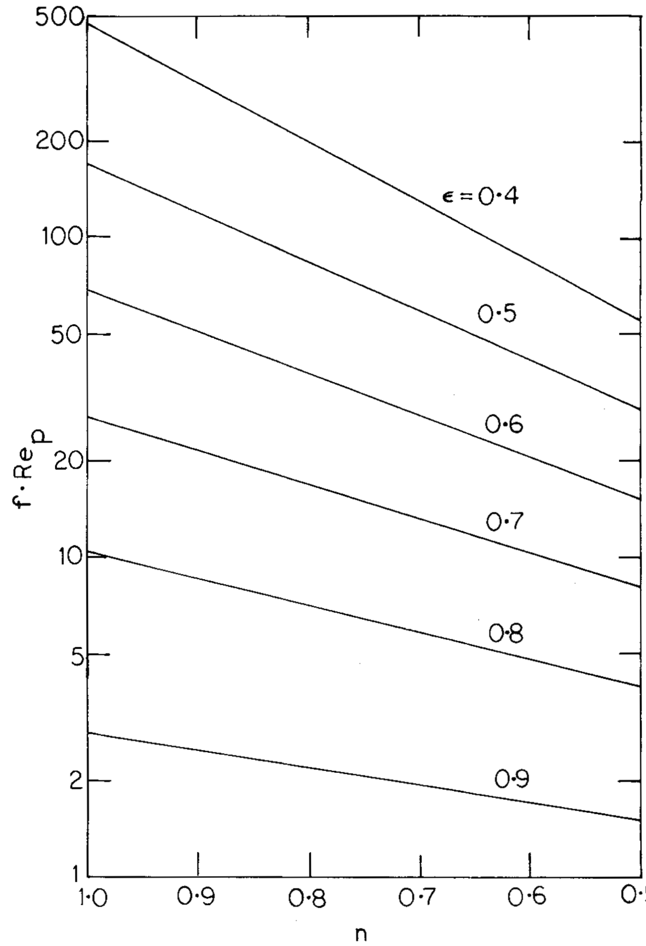

Fig. 2. Arithmetic average of the bounds on $(f.Re_p)$ vs. n for various values of ϵ.

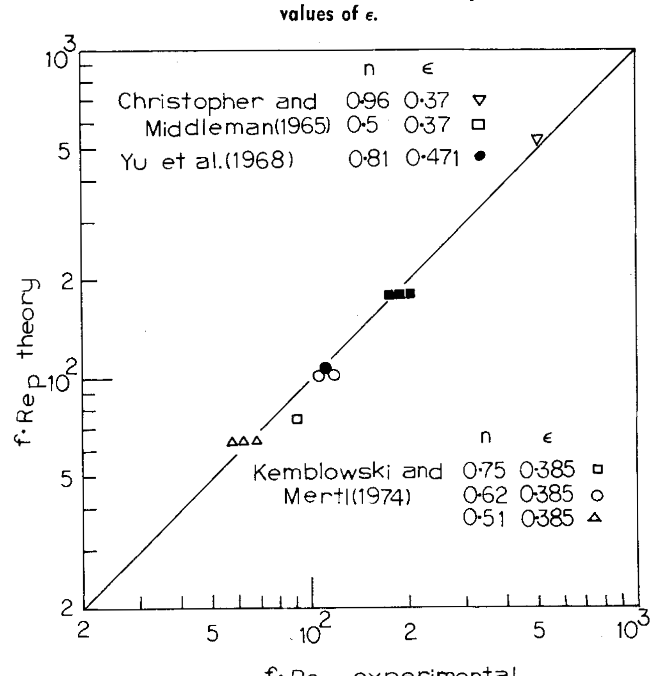

Fig. 3. Comparison of experimental and predicted values of $(f.Re_p)$.

NOTATION

a = radius of solid sphere

A_1 to A_4 = constants in the trial stream function profile

A', A'', B, C, C', C'' = constants in the trial extra stress profile
C_d = drag coefficient
d_{ij} = component of the rate-of-deformation tensor
D, E, E', E'' = constants in the trial extra stress profile
E = work function
E_c = complementary work function
$\mathcal{E}$ = energy dissipation rate
f = Fanning friction factor, $\Delta Pa/PV_o^2L$
$\mathbf{f}$ = body force
F, F', F'' = constants in the trial extra stress profile
F_d = drag force on the sphere
I = unit tensor
II = second invariant of the rate-of-deformation tensor
II_τ = second invariant of the extra stress tensor
J_1, J_2 = integrals defined by (24) and (38)
K = consistency index
n = flow behavior index
$\mathbf{n}$ = normal vector in surface integrals
p = pressure
$\mathbf{r}$ = dimensionless radius vector
$\mathbf{R}$ = radius vector
r_∞ = dimensionless radius of the free surface
R_∞ = radius of the free surface
Re_p = Reynolds number, $V_o^{2-n}(2a)^n\ \rho/K$
S = bounding surface of the flow domain
S_t = the part of the bounding surface on which the stress is explicitly stated
S_v = the part of the bounding surface on which the velocity is explicitly stated
$tr(\mathbf{A})$ = trace of matrix $\mathbf{A}$
$\mathbf{T}$ = stress tensor
$\mathbf{v}$ = velocity vector
V = volume domain of the flow
V_o = superficial velocity of the fluid in the assemblage
x = reciprocal r
x_∞ = reciprocal r_∞
Y_P = $C_dRe_p/24$
Y_{UB}, Y_{LB} = upper and lower bounds on Y_P
z = $\cos\theta$

Greek Letters

δ_{ij} = Kronecker delta
ϵ = bed voidage
η = fluid viscosity
ρ = fluid density
σ = constant in the stream function profile
$\boldsymbol{\tau}$ = extra stress tensor
ϕ = azimuthal coordinate
Φ = body force potential
ψ = stream function

Symbols

∇ = operator del
. = dot product

Subscripts

r, θ = radial and angular components

Superscripts

— = dimensionless quantity
* = quantity derived from trial profile

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A Comprehensive Correlating Equation for Forced Convection from Flat Plates

A correlating equation was developed which provides a continuous representation for all *Pr* and *Re*. Different constants are suggested for the local and mean Nusselt numbers and for uniform wall temperature and heating. These constants are based on the best available theoretical and experimental results.

STUART W. CHURCHILL

Department of Chemical and Biochemical Engineering University of Pennsylvania, Philadelphia, Pennsylvania 19174